Twisted quantum affine superalgebra $U_q[sl(2|2)^{(2)}]$, $U_q[osp(2|2)]$ invariant ***R***-matrices and a new integrable electronic model

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Twisted quantum affine superalgebra $U_q[sl(2|2)^{(2)}]$, $U_q[osp(2|2)]$ invariant R-matrices and a new integrable electronic model

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Abstract. We describe the twisted affine superalgebra $sl(2|2)^{(2)}$ and its quantized version $U_q[sl(2|2)^{(2)}]$. We investigate the tensor product representation of the four-dimensional grade star representation for the fixed-point subsuperalgebra $U_q[osp(2|2)]$. We work out the tensor product decomposition explicitly and find that the decomposition is not completely reducible. Associated with this four-dimensional grade star representation we derive two $U_q[osp(2|2)]$ invariant R-matrices: one of them corresponds to $U_q[sl(2|2)^{(2)}]$ and the other to $U_q[osp(2|2)^{(1)}]$. Using the R-matrix for $U_q[sl(2|2)^{(2)}]$, we construct a new $U_q[osp(2|2)]$ invariant strongly correlated electronic model, which is integrable in one dimension. Interestingly this model reduces in the $q = 1$ limit, to the one proposed by Essler *et al* which has a larger $sl(2|2)$ symmetry.

1. Introduction

Quantum affine algebras describe the underlying symmetries of integrable systems, conformal field theories, exactly solvable models and integrable quantum field theories. Quantum affine superalgebras are $\mathbb{Z}_2$-graded generalizations [1, 2] of the bosonic quantum algebras and are mathematical objects of importance in the study of supersymmetric theories. Examples are supersymmetric lattice models of strongly correlated electrons such as the supersymmetric t-J model [3], the extended Hubbard model [4] and the supersymmetric U model proposed in [5] and exactly solved in [6]. In each case these models are derived from an R-matrix satisfying the Yang–Baxter equation. The construction of these R-matrices can be achieved within the framework of the quantum affine superalgebras.

Despite their significance, quantum affine superalgebras have so far remained largely ignored in the literature. This is particularly the case for the twisted quantum affine superalgebras. In this paper we will study the twisted affine superalgebra $U_q[sl(2|2)^{(2)}]$ and one interesting representation for its fixed-point subsuperalgebra $U_q[osp(2|2)]$.

Lie superalgebras are much richer structures and have a more complicated representation theory than their bosonic counterparts [1, 7]. For instance, a given Lie superalgebra allows many inequivalent systems of simple roots and these give rise to different Hopf algebras

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upon deformation. As will be seen below, one has to work with the non-standard simple root system of $sl(2|2)$ to obtain the twisted superalgebra $sl(2|2)^{(2)}$.

For every pair of finite-dimensional irreps of a quantum affine superalgebra there exists a solution to the Yang–Baxter equation [8]. In a recent paper [9] we showed how to construct R-matrices for twisted bosonic quantum algebras. Our work has immediately been taken up and generalized by the authors in [10]. For the type of irrep considered in the present paper, however, care must be taken since the tensor product decomposition of two such irreps is not completely reducible. This problem is solved by introducing a nilpotent operator of order two. Using this approach, we will determine the spectral dependent R-matrices for $U_q[sl(2|2)^{(2)}]$ and $U_q[osp(2|2)^{(1)}]$.

The R-matrix for $U_q[sl(2|2)^{(2)}]$ has an interesting feature that in the rational limit it becomes $sl(2|2)$ invariant. Using this R-matrix, we will derive a new $U_q[osp(2|2)]$ invariant model of strongly correlated electrons which is integrable on a one-dimensional lattice. This model has different interaction terms from the ones in the models [3–5].

This paper is organized as follows. In sections 2 and 3, we study the twisted affine superalgebra $sl(2|2)^{(2)}$ and its quantized version $U_q[sl(2|2)^{(2)}]$, respectively. The tensor product representation of the four-dimensional grade star representation for the fixed subsuperalgebra $U_q[osp(2|2)]$ is also investigated in detail and the basis and its dual for this irrep is constructed explicitly. In section 4 we derive the R-matrix associated with the four-dimensional irrep of $U_q[sl(2|2)^{(2)}]$. Using this R-matrix, we propose, in section 5, a new model of strongly correlated electrons which is exactly solvable on a one-dimensional lattice. In section 6 we rederive the R-matrix associated with $U_q[osp(2|2)^{(1)}]$. In section 7 we give some concluding remarks.

2. Twisted affine superalgebra $sl(2|2)^{(2)}$

We recall the relevant information about twisted affine superalgebras [1, 11]. Let L be a finite-dimensional simple Lie superalgebra and τ a diagram automorphism of L of order k. Associated with these one constructs the twisted affine superalgebra $L^{(k)}$. In this paper we will assume $k = 2$. Let L_0 be the fixed-point subalgebra under the diagram automorphism σ. We recall that

$$L = L_0 \oplus L_1 \qquad [L_i, L_j] = L_{(i+j)\mathrm{mod}2}. \tag{2.1}$$

L_1 gives rise to a L_0-module under the adjoint action of L_0. Let θ_0 be its highest weight.

Let us consider $L = sl(2|2)$, whose generators we denote as E^i_j, $i, j = 1, 2, 3, 4$. We choose the grading $[1] = [4] = 0$ and $[2] = [3] = 1$. The $sl(2|2)$ generators satisfy the graded commutation relations

$$[E^i_j, E^k_l] = \delta^k_j E^i_l - (-1)^{([i]+[j])([k]+[l])}\delta^i_l E^k_j. \tag{2.2}$$

We work with the non-standard root system of $sl(2|2)$. Then the associated Dynkin diagram has an automorphism τ of order 2 [2]. Under τ the root vectors associated with this diagram, which are E^1_3, E^3_2 and E^2_4, transform in the following fashion:

$$\tau(E^1_3) = E^2_4 \qquad \tau(E^3_2) = E^3_2 \qquad \tau(E^2_4) = E^1_3. \tag{2.3}$$

This, together with relations

$$\tau(E^1_1) = -E^4_4 \qquad \tau(E^2_2) = -E^3_3 \qquad \tau(E^3_3) = -E^2_2 \qquad \tau(E^4_4) = -E^1_1 \tag{2.4}$$

leads us to define the following transformation rules for other generators in order that the graded commutation relations are invariant under the automorphism:

$$\begin{array}{llll} \tau(E_2^1) = -E_4^3 & \tau(E_4^1) = -E_4^1 & \tau(E_4^3) = -E_2^1 & \tau(E_3^2) = E_3^2 \\ \tau(E_1^3) = -E_2^4 & \tau(E_2^4) = -E_1^3 & \tau(E_1^2) = E_3^4 & \tau(E_1^4) = -E_1^4 \\ \tau(E_3^4) = E_1^2. & & & \end{array} \quad (2.5)$$

With respect to the eigenvectors of τ we have the decomposition $sl(2|2) = sl(2|2)_0 \oplus sl(2|2)_1$, where

$$\begin{aligned} sl(2|2)_0 &= \{X \in sl(2|2), \tau(X) = X\} = \{E_2^2 - E_3^3, E_3^2, E_2^3, \tfrac{1}{2}(E_3^3 + E_4^4 - E_1^1 - E_2^2), \\ &\quad \tfrac{1}{\sqrt{2}}(-E_2^1 + E_4^3), \tfrac{1}{\sqrt{2}}(E_1^2 + E_3^4), \tfrac{1}{\sqrt{2}}(E_3^1 + E_4^2), \tfrac{1}{\sqrt{2}}(-E_1^3 + E_2^4)\} \\ sl(2|2)_1 &= \{X \in sl(2|2), \tau(X) = -X\} = \{\mathrm{i}E_4^1, \mathrm{i}E_1^4, \tfrac{1}{\sqrt{2}}(E_2^1 + E_4^3), \tfrac{1}{\sqrt{2}}(-E_1^2 + E_3^4), \\ &\quad \tfrac{1}{\sqrt{2}}(-E_3^1 + E_4^2), \tfrac{1}{\sqrt{2}}(E_1^3 + E_2^4), E_1^1 + E_2^2 + E_3^3 + E_4^4\}. \end{aligned} \quad (2.6)$$

It is easily seen that the fixed-point subsuperalgebra $sl(2|2)_0$ is nothing but $osp(2|2) = sl(2|1)$.

We recall that $sl(2|2)$ admits Chevalley generators $\{E_i, F_i, H_i, i = 0, 1, 2\}$:

$$\begin{array}{llll} E_1 = E_3^2 & F_1 = E_2^3 & H_1 = E_2^2 - E_3^3 & E_2 = \frac{1}{\sqrt{2}}(-E_2^1 + E_4^3) \\ F_2 = \frac{1}{\sqrt{2}}(E_1^2 + E_3^4) & H_2 = \frac{1}{2}(E_3^3 + E_4^4 - E_1^1 - E_2^2) & E_0 = \mathrm{i}E_1^4 & \\ F_0 = \mathrm{i}E_4^1 & H_0 = E_1^1 - E_4^4. & & \end{array} \quad (2.7)$$

Here E_i, F_i, H_i, $i = 1, 2$, form the Chevalley generators for $sl(2|2)_0$. $E_0 \in sl(2|2)_1$ corresponds to the minimal weight vector and thus has weight $-\theta_0$. It follows that $H_0 = -n_1 H_1 - n_2 H_2$ lies in the Cartan subalgebra H of $sl(2|2)_0$. The integers n_1, n_2 are known as the Kac labels of $sl(2|2)^{(2)}$.

We now introduce the corresponding twisted affine superalgebra $sl(2|2)^{(2)}$ which admits the decomposition

$$sl(2|2)^{(2)} = \bigoplus_{m \in \frac{1}{2}\mathbb{Z}} L_m \oplus \mathbb{C}c_0 \qquad L_m = \begin{cases} L_0(m), m \in \mathbb{Z} \\ L_1(m), m \in \mathbb{Z} + \frac{1}{2} \end{cases} \quad (2.8)$$

with $L_a(m) = \{X(m) | x \in L_a\}$, $a = 0, 1$ and c_0 a central charge. The graded Lie bracket is given by

$$[X(m), Y(n)] = [X, Y](m + n) + m c_0 \delta_{m+n,0}(X, Y) \qquad [c_0, X(m)] = 0. \quad (2.9)$$

Here $(,)$ is the fixed invariant bilinear form on $sl(2|2)$. A suitable set of generators for $sl(2|2)^{(2)}$ is given by

$$\begin{array}{llll} e_i = E_i(0) & h_i = H_i(0) & f_i = F_i(0) & i = 1, 2 \\ e_0 = E_0(1/2) & h_0 = H_0(0) + c_0/2 & f_0 = F_0(-1/2). & \end{array} \quad (2.10)$$

These simple generators satisfy the defining relations of $sl(2|2)^{(2)}$:

$$\begin{array}{lll} [e_i, f_j] = \delta_{ij} h_i & e_2^2 = 0 = f_2^2 & \\ [h_0, e_0] = -2e_0 & [h_0, e_1] = 0 & [h_0, e_2] = e_2 \\ [h_1, e_0] = 0 & [h_1, e_1] = 2e_1 & [h_1, e_2] = -e_2 \\ [h_2, e_0] = e_0 & [h_2, e_1] = -e_1 & [h_2, e_2] = 0 \\ [h_0, f_0] = 2f_0 & [h_0, f_1] = 0 & [h_0, f_2] = -f_2 \end{array}$$

$$[h_1, f_0] = 0 \qquad [h_1, f_1] = -2f_1 \qquad [h_1, f_2] = f_2$$
$$[h_2, f_0] = -f_0 \qquad [h_2, f_1] = f_1 \qquad [h_2, f_2] = 0$$
$$(\mathrm{ad} e_1)^2 e_2 = (\mathrm{ad} e_0) e_1 = (\mathrm{ad} e_0)^2 e_2 = 0$$
$$(\mathrm{ad} f_1)^2 f_2 = (\mathrm{ad} f_0) f_1 = (\mathrm{ad} f_0)^2 f_2 = 0. \tag{2.11}$$

We have an algebra homomorphism, called the *evaluation map*, $ev_x : U[sl(2|2)^{(2)}] \to \mathbb{C}[x, x^{-1}] \otimes U[sl(2|2)]$, with $U[sl(2|2)^{(2)}]$, $U[sl(2|2)]$ the enveloping algebras of $sl(2|2)^{(2)}$, $sl(2|2)$ respectively, given by

$$ev_x(X(m)) = x^{2m} X \qquad ev_x(c_0) = 0 \tag{2.12}$$

and extended to all of $U[sl(2|2)^{(2)}]$ in the natural way. Thus given a finite-dimensional $sl(2|2)$-module V carrying a representation π we have a corresponding $sl(2|2)^{(2)}$ module $V(x) = \mathbb{C}[x, x^{-1}] \otimes V$ carrying the *loop representation* $\hat{\pi}$ given by

$$\hat{\pi} = (1 \otimes \pi) ev_x. \tag{2.13}$$

Below we will see how such representations of $osp(2|2)$ can be quantized to give solutions of the Yang–Baxter equation.

3. $U_q[sl(2|2)^{(2)}]$

Corresponding to the twisted affine algebra $sl(2|2)^{(2)}$ we have the twisted quantum affine algebra $U_q[sl(2|2)^{(2)}]$ with generators $q^{\pm h_i/2}$, e_i, f_i, $(i = 0, 1, 2)$ and defining relations

$$[e_i, f_j] = \delta_{ij} \frac{q^{h_i} - q^{-h_i}}{q - q^{-1}} \qquad e_2^2 = 0 = f_2^2$$
$$q^{h_0} e_0 q^{-h_0} = q^{-2} e_0 \qquad q^{h_0} e_1 q^{-h_0} = e_1 \qquad q^{h_0} e_2 q^{-h_0} = q e_2$$
$$q^{h_1} e_0 q^{-h_1} = e_0 \qquad q^{h_1} e_1 q^{-h_1} = q^2 e_1 \qquad q^{h_1} e_2 q^{-h_1} = q^{-1} e_2$$
$$q^{h_2} e_0 q^{-h_2} = q e_0 \qquad q^{h_2} e_1 q^{-h_2} = q^{-1} e_1 \qquad q^{h_2} e_2 q^{-h_2} = e_2$$
$$q^{h_0} f_0 q^{-h_0} = q^2 f_0 \qquad q^{h_0} f_1 q^{-h_0} = f_1 \qquad q^{h_0} f_2 q^{-h_0} = q^{-1} f_2$$
$$q^{h_1} f_0 q^{-h_1} = f_0 \qquad q^{h_1} f_1 q^{-h_1} = q^{-2} f_1 \qquad q^{h_1} f_2 q^{-h_1} = q f_2$$
$$q^{h_2} f_0 q^{-h_2} = q^{-1} f_0 \qquad q^{h_2} f_1 q^{-h_2} = q f_1 \qquad q^{h_2} f_2 q^{-h_2} = f_2$$
$$e_0 e_1 - e_1 e_0 = 0 \qquad e_0^2 e_2 + e_2 e_0^2 - (q + q^{-1}) e_0 e_2 e_0 = 0$$
$$e_1^2 e_2 + e_2 e_1^2 - (q + q^{-1}) e_1 e_2 e_1 = 0$$
$$f_0 f_1 - f_1 f_0 = 0 \qquad f_0^2 f_2 + f_2 f_0^2 - (q + q^{-1}) f_0 f_2 f_0 = 0$$
$$f_1^2 f_2 + f_2 f_1^2 - (q + q^{-1}) f_1 f_2 f_1 = 0. \tag{3.1}$$

Throughout this paper we will assume that q is generic, i.e. not a root of unity and $[n]_q = (q^n - q^{-n})/(q - q^{-1})$.

The algebra $U_q[sl(2|2)^{(2)}]$ is a Hopf algebra. The coproduct is given by

$$\Delta(q^{\pm h}) = q^{\pm h} \otimes q^{\pm h} \qquad \Delta(e_i) = e_i \otimes q^{-h_i/2} + q^{h_i/2} \otimes e_i$$
$$\Delta(f_i) = f_i \otimes q^{-h_i/2} + q^{h_i/2} \otimes f_i. \tag{3.2}$$

We omit the formulae for the antipode and the counit. The multiplication rule for the tensor product is defined for elements $a, b, c, d \in U_q[sl(2|2)^{(2)}]$ by

$$(a \otimes b)(c \otimes d) = (-1)^{[b][c]} (ac \otimes bd). \tag{3.3}$$

The (minimal) four-dimensional irreducible representation of $U_q[sl(2|2)]$ is undeformed. That is the representation matrices for the fundamental generators are the same as in the classical case. Choosing a basis $|4\rangle = (0,0,0,1)^t$, $|3\rangle = (0,0,1,0)^t$, $|2\rangle = (0,1,0,0)^t$, $|1\rangle = (1,0,0,0)^t$, with $|1\rangle, |4\rangle$ even (bosonic) and $|2\rangle, |3\rangle$ odd (fermionic), the representation matrices are $E^i_j = e^i_j$, where $(e^i_j)^k_l = \delta^{ik}\delta_{jl}$. Using the $U_q[sl(2|2)]$ generators $\{E_i, F_i, H_i, i = 0, 1, 2\}$ this representation is written as

$$
\begin{aligned}
&E_1 = e^2_3 \qquad F_1 = e^3_2 \qquad H_1 = e^2_2 - e^3_3 \\
&E_2 = \sqrt{[1/2]_q}(-e^1_2 + e^3_4) \qquad F_2 = \sqrt{[1/2]_q}(e^2_1 + e^4_3) \qquad H_2 = \tfrac{1}{2}(e^3_3 + e^4_4 - e^1_1 - e^2_2) \\
&E_0 = \mathrm{i}e^4_1 \qquad F_0 = \mathrm{i}e^1_4 \qquad H_0 = e^1_1 - e^4_4.
\end{aligned} \tag{3.4}
$$

It can be shown that there exists an evaluation representation of $U_q[sl(2|2)^{(2)}]$ given by

$$
\begin{aligned}
&e_i = E_i \qquad f_i = F_i \qquad h_i = H_i \qquad i = 1, 2 \qquad e_0 = xE_0 \qquad f_0 = x^{-1}F_0 \\
&\qquad h_0 = H_0.
\end{aligned} \tag{3.5}
$$

The four-dimensional representation of $U_q[sl(2|2)]$ is also irreducible under the $U_q[osp(2|2)]$ subsuperalgebra. We call such a representation $U_q[osp(2|2)]$-irreducible. Equation (3.5) implies that this irreducible four-dimensional $U_q[osp(2|2)]$-module, denoted as V in what follows, is affinizable to also provide an irreducible $U_q[sl(2|2)^{(2)}]$ representation. As in the classical case [7], the tensor product of two such $U_q[osp(2|2)]$-irreducible representations is not completely reducible. This can be seen as follows. Introduce the graded permutation operator P on the tensor product module $V \otimes V$ such that

$$P(v_\alpha \otimes v_\beta) = (-1)^{[\alpha][\beta]} v_\beta \otimes v_\alpha \qquad \forall v_\alpha, v_\beta \in V. \tag{3.6}$$

We decompose the tensor product as

$$V \otimes V = W_+ \oplus W_- \tag{3.7}$$

with $W\pm$ being eigenspaces of P in the $q = 1$ limit

$$W_\pm = \left\{ v \in V \otimes V \,|\, \lim_{q\to 1}(P \mp 1)v = 0 \right\}. \tag{3.8}$$

It is easy to check that the states

$$
\begin{aligned}
|\psi^-_1\rangle &= \frac{1}{\sqrt{q^{1/2} + q^{-1/2}}}(q^{1/4}|1\rangle \otimes |2\rangle - q^{-1/4}|2\rangle \otimes |1\rangle) \\
|\psi^-_2\rangle &= \frac{1}{\sqrt{q^{1/2} + q^{-1/2}}}(q^{1/4}|1\rangle \otimes |3\rangle - q^{-1/4}|3\rangle \otimes |1\rangle) \\
|\psi^-_3\rangle &= |2\rangle \otimes |2\rangle \\
|z\rangle &= \frac{1}{2}(|1\rangle \otimes |4\rangle - |4\rangle \otimes |1\rangle + |2\rangle \otimes |3\rangle + |3\rangle \otimes |2\rangle) \\
|w\rangle &= \frac{1}{\sqrt{q + q^{-1}}}(q^{1/2}|2\rangle \otimes |3\rangle + q^{-1/2}|3\rangle \otimes |2\rangle) \\
|\psi^-_6\rangle &= |3\rangle \otimes |3\rangle \\
|\psi^-_7\rangle &= \frac{1}{\sqrt{q^{1/2} + q^{-1/2}}}(q^{1/4}|2\rangle \otimes |4\rangle - q^{-1/4}|4\rangle \otimes |2\rangle) \\
|\psi^-_8\rangle &= \frac{1}{\sqrt{q^{1/2} + q^{-1/2}}}(q^{1/4}|3\rangle \otimes |4\rangle - q^{-1/4}|4\rangle \otimes |3\rangle)
\end{aligned} \tag{3.9}
$$

span the invariant subspace W_-, and we set

$$\langle\psi^-| = (|\psi^-\rangle)^\dagger \qquad |\psi^-\rangle = |\psi^-_k\rangle, |z\rangle, |w\rangle \tag{3.10}$$

where

$$(|\beta\rangle \otimes |\gamma\rangle)^\dagger = (-1)^{[|\beta\rangle][|\gamma\rangle]}(|\beta\rangle)^\dagger \otimes (|\gamma\rangle)^\dagger$$
$$(|\beta\rangle)^\dagger = \langle\beta| \qquad \forall\beta = 1, 2, 3, 4. \tag{3.11}$$

Notice that the states $|z\rangle$ and $|w\rangle$ are not orthonormal to each other. The remaining states are combined as follows

$$|\psi_1^+\rangle = |1\rangle \otimes |1\rangle$$
$$|\psi_2^+\rangle = \frac{1}{\sqrt{q^{1/2}+q^{-1/2}}}(q^{-1/4}|1\rangle \otimes |2\rangle + q^{1/4}|2\rangle \otimes |1\rangle)$$
$$|\psi_3^+\rangle = \frac{1}{\sqrt{q^{1/2}+q^{-1/2}}}(q^{-1/4}|1\rangle \otimes |3\rangle + q^{1/4}|3\rangle \otimes |1\rangle)$$
$$|s\rangle = \frac{1}{\sqrt{2(q+q^{-1})}}(q^{-1/2}|1\rangle \otimes |4\rangle + q^{1/2}|4\rangle \otimes |1\rangle + q^{-1/2}|2\rangle \otimes |3\rangle - q^{1/2}|3\rangle \otimes |2\rangle)$$
$$|c\rangle = \frac{1}{\sqrt{2(q^2+1)(3-2q+3q^2)}}((2q^2-q+1)|1\rangle \otimes |4\rangle + (q^2-q+2)|4\rangle \otimes |1\rangle$$
$$-(q+1)|2\rangle \otimes |3\rangle + q(q+1)|3\rangle \otimes |2\rangle)$$
$$|\psi_6^+\rangle = \frac{1}{\sqrt{q^{1/2}+q^{-1/2}}}(q^{-1/4}|2\rangle \otimes |4\rangle + q^{1/4}|4\rangle \otimes |2\rangle)$$
$$|\psi_7^+\rangle = \frac{1}{\sqrt{q^{1/2}+q^{-1/2}}}(q^{-1/4}|3\rangle \otimes |4\rangle + q^{1/4}|4\rangle \otimes |3\rangle)$$
$$|\psi_8^+\rangle = |4\rangle \otimes |4\rangle \tag{3.12}$$

where, as above, we have used $\langle\psi^+| = (|\psi^+\rangle)^\dagger$, where $|\psi^+\rangle$ stands for $|\psi_k^+\rangle$, $|s\rangle$, $|c\rangle$.

One can show that: (i) the eight states (3.2) span the invariant subspace W_+ and $|c\rangle$ is a cyclic vector for the corresponding representation; (ii) $|s\rangle$ spans a one-dimensional invariant subspace, i.e. it is mapped into zero by all generators of $U_q[osp(2|2)]$. However, the singlet state $|s\rangle$ is not separable from the representation. Therefore the tensor product is not completely reducible.

Recall that $|z\rangle$, $|w\rangle$, $|s\rangle$ and $|c\rangle$ are not orthonormal to each other. Let us construct the dual of these states. Denote

$$|\psi_1\rangle \equiv |z\rangle \qquad |\psi_2\rangle \equiv |w\rangle \qquad |\psi_3\rangle \equiv |s\rangle \qquad |\psi_4\rangle \equiv |c\rangle \tag{3.13}$$

and define a metric g_{ij}:

$$g_{ij} = \langle\psi_i|\psi_j\rangle \qquad i, j = 1, 2, 3, 4. \tag{3.14}$$

It is easily shown that

$$g_{11} = g_{22} = g_{33} = g_{44} = 1 \qquad g_{12} = g_{21} = \frac{q^{1/2}+q^{-1/2}}{2\sqrt{q+q^{-1}}}$$
$$g_{13} = g_{31} = \frac{q^{-1/2}-q^{1/2}}{\sqrt{2(q+q^{-1})}} \qquad g_{23} = g_{32} = g_{24} = g_{42} = 0$$
$$g_{34} = g_{43} = 0 \qquad g_{14} = g_{41} = \frac{q^2-1}{\sqrt{2(q^2+1)(3-2q+3q^2)}}. \tag{3.15}$$

We can define dual states as follows

$$\langle\psi^i| = g^{ij}\langle\psi_j| \qquad (g^{ij}) = (g_{ij})^{-1} \tag{3.16}$$

where summation on the repeated index j is implied. A long exercise leads to

$$
\begin{aligned}
\langle\psi^1| &= \frac{2}{(1+q)^2}((1+q^2)(\langle 1|\otimes\langle 4| - \langle 4|\otimes\langle 1|) + (1-q)(\langle 2|\otimes\langle 3| - q\langle 3|\otimes\langle 2|)) \\
\langle\psi^2| &= \frac{\sqrt{1+q^2}}{1+q}(-\langle 1|\otimes\langle 4| + \langle 4|\otimes\langle 1| - \langle 2|\otimes\langle 3| - \langle 3|\otimes\langle 2|) \\
\langle\psi^3| &= \frac{1}{(1+q)^2\sqrt{2(q^2+1)}}((-1+4q-q^2+2q^3)\langle 1|\otimes\langle 4| \\
&\quad +(2-q+4q^2-q^3)\langle 4|\otimes\langle 1| - (3-2q+3q^2)(\langle 2|\otimes\langle 3| - q\langle 3|\otimes\langle 2|)) \\
\langle\psi^4| &= \sqrt{\frac{3-2q+3q^2}{2(q^2+1)}}\frac{1}{1+q}(\langle 1|\otimes\langle 4| + q\langle 4|\otimes\langle 1| + \langle 2|\otimes\langle 3| - q\langle 3|\otimes\langle 2|).
\end{aligned}
\tag{3.17}
$$

We remark that $\langle\psi^4|$ spans a one-dimensional right submodule under the quantum group action.

4. *R*-matrix for $U_q[sl(2|2)^{(2)}]$

With an abuse of notation, in this section we set $e_0 = \mathrm{i}e_1^4$, $f_0 = \mathrm{i}e_4^1$ and $h_0 = e_1^1 - e_4^4$. It can be shown [8] that a solution to the linear equations

$$
\begin{aligned}
&R(x)\Delta(a) = \bar{\Delta}(a)R(x) \qquad \forall a \in U_q[osp(2|2)] \\
&R(x)(xe_0\otimes q^{-h_0/2} + q^{h_0/2}\otimes e_0) = (xe_0\otimes q^{h_0/2} + q^{-h_0/2}\otimes e_0)R(x)
\end{aligned}
\tag{4.1}
$$

satisfies the QYBE

$$
R_{12}(x)R_{13}(xy)R_{23}(y) = R_{23}(y)R_{13}(xy)R_{12}(x). \tag{4.2}
$$

In the above, $\bar{\Delta} = T\cdot\Delta$, with T the twist map defined by $T(a\otimes b) = (-1)^{[a][b]}b\otimes a$, $\forall a, b \in U_q[osp(2|2)]$ and also, if $R(x) = \sum_i a_i\otimes b_i$, then $R_{12}(x) = \sum_i a_i\otimes b_i\otimes I$ etc. The solution to (4.1) is unique, up to scalar functions. The multiplicative spectral parameter x can be transformed into an additive spectral parameter u by $x = \exp(u)$.

In all our equations we implicitly use the 'graded' multiplication rule (3.3). Thus the R-matrix of a quantum superalgebra satisfies a 'graded' QYBE which, when written as an ordinary matrix equation, contains extra signs:

$$
\begin{aligned}
&R(x)_{ij}^{i'j'}R(xy)_{i'k}^{i''k'}R(y)_{j'k'}^{j''k''}(-1)^{[i][j]+[k][i']+[k'][j']} \\
&\qquad = R(y)_{jk}^{j'k'}R(xy)_{ik'}^{i'k''}R(x)_{i'j'}^{i''j''}(-1)^{[j][k]+[k'][i]+[j'][i']}.
\end{aligned}
\tag{4.3}
$$

However after a redefinition

$$
\tilde{R}(\cdot)_{ij}^{i'j'} = R(\cdot)_{ij}^{i'j'}(-1)^{[i][j]} \tag{4.4}
$$

the signs disappear from the equation. Thus any solution of the 'graded' QYBE arising from the R-matrix of a quantum superalgebra provides also a solution of the standard QYBE after the redefinition in (4.4).

Set

$$
\check{R}(x) = PR(x) \tag{4.5}
$$

where P is the graded permutation operator on $V\otimes V$. Then (4.1) can be rewritten as

$$
\begin{aligned}
&\check{R}(x)\Delta(a) = \Delta(a)\check{R}(x) \qquad \forall a \in U_q[osp(2|2)] \\
&\check{R}(x)(xe_0\otimes q^{-h_0/2} + q^{h_0/2}\otimes e_0) = (e_0\otimes q^{-h_0/2} + xq^{h_0/2}\otimes e_0)\check{R}(x)
\end{aligned}
\tag{4.6}
$$

and in terms of $\check{R}(x)$ the QYBE becomes

$$(I \otimes \check{R}(x))(\check{R}(xy) \otimes I)(I \otimes \check{R}(y)) = (\check{R}(y) \otimes I)(I \otimes \check{R}(xy))(\check{R}(x) \otimes I). \tag{4.7}$$

Note that this equation, if written in matrix form, does not have extra signs. This is because the definition of the graded permutation operator in (3.6) includes the signs of (4.4). In the following we will normalize the R-matrix $\check{R}(x)$ in such a way that $\check{R}(x)\check{R}(x^{-1}) = I$, which is usually called the unitarity condition in the literature.

Let us proceed to solve $\check{R}(x)$ satisfying (4.6) for $U_q[sl(2|2)^{(2)}]$, that is for $e_0 = \mathrm{i}e_1^4, h_0 = e_1^1 - e_4^4$. As we have shown in the last section, the tensor product decomposition is not completely reducible. Therefore the tensor product graph method developled in [9, 10] is not applicable to the present case. Let $P[\pm]$ denote the (central) projection operators defined by

$$P[\pm](V \otimes V) = W_\pm \tag{4.8}$$

and N the operator mapping the cyclic vector of $V \otimes V$ to the singlet $V_0 \subset W_+ \subset V \otimes V$. Obviously N is nilpotent of order 2 (i.e. $N^2 = 0$). Using the states from the section, $P[\pm]$ and N can be expressed as

$$\begin{aligned}
P[+] &= |\psi_1^+\rangle\langle\psi_1^+| + |\psi_2^+\rangle\langle\psi_2^+| + |\psi_3^+\rangle\langle\psi_3^+| + |\psi_6^+\rangle\langle\psi_6^+| + |\psi_7^+\rangle\langle\psi_7^+| + |\psi_8^+\rangle\langle\psi_8^+| \\
&\quad + |\psi_3\rangle\langle\psi^3| + |\psi_4\rangle\langle\psi^4| \\
P[-] &= |\psi_1^-\rangle\langle\psi_1^-| + |\psi_2^-\rangle\langle\psi_2^-| + |\psi_3^-\rangle\langle\psi_3^-| + |\psi_6^-\rangle\langle\psi_6^-| + |\psi_7^-\rangle\langle\psi_7^-| + |\psi_8^-\rangle\langle\psi_8^-| \\
&\quad + |\psi_1\rangle\langle\psi^1| + |\psi_2\rangle\langle\psi^2| \\
N &= f(q)|\psi_3\rangle\langle\psi^4|
\end{aligned} \tag{4.9}$$

where $f(q)$ is an arbitrary factor depending on q. It is worth pointing out that $P[\pm]$ and N are all quantum group $U_q[osp(2|2)]$ invariants. Moreover they satisfy the following relations

$$\begin{aligned}
&P[\pm]P[\pm] = P[\pm] \qquad N^2 = 0 \qquad P[+]P[-] = P[-]P[+] = 0 \\
&P[+]N = NP[+] = N \qquad P[-]N = NP[-] = 0 \\
&P[+] + P[-] = 1.
\end{aligned} \tag{4.10}$$

With the help of these operators, the most general $\check{R}(x)$ satisfying the first equation in (4.6) may be written in the form

$$\check{R}(x) = \rho_+(x)P[+] + \rho_N(x)N + \rho_-(x)P[-] \tag{4.11}$$

where $\rho_\pm(x)$, $\rho_N(x)$, are unknown functions depending on x, q.

Multiplying the second equation in (4.6) by $P[+]$ from the left and the resulting equation by $P[+]$ from the right, one gets

$$\begin{aligned}
&(\rho_+(x)P[+] + \rho_N(x)N)(xe_0 \otimes q^{-h_0/2} + q^{h_0/2} \otimes e_0)P[+] \\
&\qquad = P[+](e_0 \otimes q^{-h_0/2} + xq^{h_0/2} \otimes e_0)(\rho_+(x)P[+] + \rho_N(x)N)
\end{aligned} \tag{4.12}$$

where (4.10) has been used. With the help of (4.9), (3.9), (3.12) and (3.17), one obtains from the above equation

$$\rho_N(x) = \frac{1-q}{f(q)} \frac{2(q+q^{-1})}{\sqrt{3-2q+3q^2}} \frac{1-x}{1+x} \rho_+(x). \tag{4.13}$$

If one multiplies the second equation in (4.6) by $P[-]$ from the left and the resulting equation by $P[+]$ from the right, one has

$$\begin{aligned}
&\rho_-(x)P[-](xe_0 \otimes q^{-h_0/2} + q^{h_0/2} \otimes e_0)P[+] \\
&\qquad = P[-](e_0 \otimes q^{-h_0/2} + xq^{h_0/2} \otimes e_0)(\rho_+(x)P[+] + \rho_N(x)N)
\end{aligned} \tag{4.14}$$

which gives rise to

$$\rho_-(x) = \frac{1-xq}{x-q}\rho_+(x). \tag{4.15}$$

It follows that

$$\check{R}(x) = P[+] + \frac{1-q}{f(q)}\frac{2(q+q^{-1})}{\sqrt{3-2q+3q^2}}\frac{1-x}{1+x}N + \frac{1-xq}{x-q}P[-]. \tag{4.16}$$

Remember that the arbitrary factor $f(q)$ in (4.16) cancels out with the same factor $f(q)$ appearing in the definition of N.

An interesting feature of this $U_q[osp(2|2)]$ invariant R-matrix is that in the rational limit the N term disappears from $\check{R}(x)$ and the resultant rational R-matrix becomes $sl(2|2)$ invariant: the 36-vertex model reduces to a 28-vertex one in the rational limit!

We would like to point out that Deguchi *et al* [12] obtained a similar $U_q[osp(2|2)]$ invariant R-matrix using a different approach. However, the Deguchi *et al* R-matrix appears to be different from ours in the form of the nilpotent operator. This might be because we are using a different simple root system for $osp(2|2)$. As is well known, a fixed superalgebra allows many inequivalent systems of simple roots and these give rise to different Hopf algebras upon deformation. It is easily seen that in this paper we are using a standard simple root system for $U_q[osp(2|2)]$, while Deguchi *et al* used a non-standard one. We have also shown here that the R-matrix actually comes from the twisted quantum affine superalgebra $U_q[sl(2|2)^{(2)}]$.

5. New $U_q[osp(2|2)]$ invariant electronic model

In this section we propose a new $U_q[osp(2|2)]$ invariant strongly correlated electronic model on the unrestricted 4^L-dimensional electronic Hilbert space $\otimes_{n=1}^{L}\mathbb{C}^4$, where L is the lattice length. This model has different interaction terms from previous ones introduced in [3–5].

We recall that electrons on a lattice are described by canonical Fermi operators $c_{i,\sigma}$ and $c_{i,\sigma}^\dagger$ satisfying the anti-commutation relations given by $\{c_{i,\sigma}^\dagger, c_{j,\tau}\} = \delta_{ij}\delta_{\sigma\tau}$, where $i, j = 1, 2, \ldots, L$ and $\sigma, \tau = \uparrow, \downarrow$. The operator $c_{i,\sigma}$ annihilates an electron of spin σ at site i, which implies that the Fock vacuum $|0\rangle$ satisfies $c_{i,\sigma}|0\rangle = 0$. At a given lattice site i there are four possible electronic states:

$$|0\rangle \qquad |\uparrow\rangle_i = c_{i,\uparrow}^\dagger|0\rangle \qquad |\downarrow\rangle_i = c_{i,\downarrow}^\dagger|0\rangle \qquad |\uparrow\downarrow\rangle_i = c_{i,\downarrow}^\dagger c_{i,\uparrow}^\dagger|0\rangle. \tag{5.1}$$

By $n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}$ we denote the number operator for electrons with spin σ on site i, and we write $n_i = n_{i,\uparrow} + n_{i,\downarrow}$. The spin operators S, $S^\dagger$, S^z, (in the following, the global operator $\mathcal{O}$ will be always expressed in terms of the local one $\mathcal{O}_i$ as $\mathcal{O} = \sum_{i=1}^{L}\mathcal{O}_i$ in one dimension)

$$S_i = c_{i,\uparrow}^\dagger c_{i,\downarrow} \qquad S_i^\dagger = c_{i,\downarrow}^\dagger c_{i,\uparrow} \qquad S_i^z = \tfrac{1}{2}(n_{i,\downarrow} - n_{i,\uparrow}) \tag{5.2}$$

form an $sl(2)$ algebra and they commute with the Hamiltonians that we consider below.

Using the R-matrix (4.16) and denoting

$$\check{R}_{i,i+1}(x) = I \otimes \cdots I \otimes \underbrace{\check{R}(x)}_{i\,i+1} \otimes I \otimes \cdots \otimes I \tag{5.3}$$

one may define the local Hamiltonian

$$H_{i,i+1} = \frac{\mathrm{d}}{\mathrm{d}x}\check{R}_{i,i+1}(x)\bigg|_{x=1}. \tag{5.4}$$

By (4.9), (3.9), (3.12) and (3.17) and choosing

$$|4\rangle \equiv |0\rangle \qquad |3\rangle \equiv |\downarrow\rangle \qquad |2\rangle \equiv |\uparrow\rangle \qquad |1\rangle \equiv |\uparrow\downarrow\rangle \tag{5.5}$$

one gets, after tedious but straightforward manipulation,

$$\begin{aligned}
H &\equiv \sum_{\langle i,j\rangle} H_{i,j} \\
H_{i,j} &= -c^\dagger_{i,\uparrow} c_{j,\uparrow}\left[1 - n_{i,\downarrow} - n_{j,\downarrow} - \frac{1}{2}(q^{1/2} - q^{-1/2})(n_{i,\downarrow}(1-n_{j,\downarrow}) + n_{j,\downarrow}(1-n_{i,\downarrow}))\right] \\
&\quad + c^\dagger_{i,\downarrow} c_{j,\downarrow}\Big[1 - n_{i,\uparrow} - n_{j,\uparrow} - \frac{1}{2}(q^{1/2} - q^{-1/2}) \\
&\quad \times (q(1-n_{i,\uparrow})n_{j,\uparrow} + q^{-1} n_{i,\uparrow}(1-n_{j,\uparrow}))\Big] \\
&\quad + c^\dagger_{j,\uparrow} c_{i,\uparrow}\Big[1 - n_{i,\downarrow} - n_{j,\downarrow} + \frac{1}{2}(q^{1/2} - q^{-1/2}) \\
&\quad \times (n_{i,\downarrow}(1-n_{j,\downarrow}) + n_{j,\downarrow}(1-n_{i,\downarrow}))\Big] \\
&\quad + c^\dagger_{j,\downarrow} c_{i,\downarrow}\Big[1 - n_{i,\uparrow} - n_{j,\uparrow} + \frac{1}{2}(q^{1/2} - q^{-1/2}) \\
&\quad \times (q(1-n_{i,\uparrow})n_{j,\uparrow} + q^{-1} n_{i,\uparrow}(1-n_{j,\uparrow}))\Big] \\
&\quad + \frac{1}{2}(q^{1/2} + q^{-1/2})(S^\dagger_i S_j + S^\dagger_j S_i - q^{-1} n_{i,\uparrow} n_{j,\downarrow} - q n_{i,\downarrow} n_{j,\uparrow}) \\
&\quad - \frac{1}{2}(q^{1/2} + q^{-1/2})(c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow} c_{j,\downarrow} c_{j,\uparrow} + \text{h.c.} + (q - q^{-1}) n_{i,\uparrow} n_{j,\uparrow}(n_{j,\downarrow} - n_{i,\downarrow})) \\
&\quad + \frac{q^{-2} - 2q - 3}{2(q^{1/2} + q^{-1/2})} n_{i,\uparrow} n_{i,\downarrow} + \frac{q^2 - 2q^{-1} - 3}{2(q^{1/2} + q^{-1/2})} n_{j,\uparrow} n_{j,\downarrow} \\
&\quad + q^{1/2}(n_{i,\uparrow} + n_{i,\downarrow}) + q^{-1/2}(n_{j,\uparrow} + n_{j,\downarrow})
\end{aligned} \tag{5.6}$$

where $\langle i, j\rangle$ denote nearest-neighour links on the lattice. In deriving (5.6), use has been made of the following identities

$$\begin{aligned}
&|0\rangle\langle 0| + |\downarrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\uparrow| + |\uparrow\downarrow\rangle\langle\uparrow\downarrow| = 1 \qquad |\uparrow\downarrow\rangle\langle\uparrow\downarrow| = n_\uparrow n_\downarrow \\
&|\uparrow\rangle\langle\uparrow| = n_\uparrow - n_\uparrow n_\downarrow \qquad |\downarrow\rangle\langle\downarrow| = n_\downarrow - n_\uparrow n_\downarrow.
\end{aligned} \tag{5.7}$$

Our Hamiltonian is supersymmetric and the supersymmetry algebra is $U_q[osp(2|2)]$. The global Hamiltonian commutes with global number operators of spin up and spin down, respectively. Moreover, the model is exactly solvable on the one-dimensional lattice.

In the $q = 1$ limit, our model reduces to one proposed by Essler *et al* [4] which has a larger, $sl(2|2)$, symmetry.

6. $U_q[osp(2|2)^{(1)}]$ *R*-matrix revisited

The four-dimensional grade star irrep of $U_q[osp(2|2)]$ can also be extended to carry an irreducible representation of the untwisted quantum affine superalgebra $U_q[osp(2|2)^{(1)}]$. In this case e_0 and f_0 are odd and given by

$$\begin{aligned}
&e_0 = \sqrt{[1/2]_q}(-e^3_1 + e^4_2) \qquad f_0 = -\sqrt{[1/2]_q}(e^1_3 + e^2_4) \\
&h_0 = -\tfrac{1}{2}(e^2_2 + e^4_4 - e^1_1 - e^3_3).
\end{aligned} \tag{6.1}$$

Denote the R-matrix in the present case by $\check{R}_{\rm ut}(x)$. In principal, this R-matrix can be obtained by carefully taking the $\alpha = -\frac{1}{2}$ limit of the corresponding R-matrix found in [13]. Here we rederive it more rigorously.

With the explicit expression (6.1) of e_0, f_0 and h_0, and writing the most general $\check{R}_{\rm ut}(x)$ as the form

$$\check{R}_{\rm ut}(x) = \varrho_+(x)P[+] + \varrho_N(x)N + \varrho_-(x)P[-] \tag{6.2}$$

the Jimbo equations

$$\begin{aligned}&\check{R}_{\rm ut}(x)\Delta(a) = \Delta(a)\check{R}_{\rm ut}(x) \qquad \forall a \in U_q[osp(2|2)]\\ &\check{R}_{\rm ut}(x)(xe_0 \otimes q^{-h_0/2} + q^{h_0/2} \otimes e_0) = (e_0 \otimes q^{-h_0/2} + xq^{h_0/2} \otimes e_0)\check{R}_{\rm ut}(x)\end{aligned} \tag{6.3}$$

can be solved by direct computations, as we did in the previous section. Here we proceed a bit differently. We recall that the braid generator σ, which satisfies the first equation in (6.3) and the relation

$$\sigma(e_0 \otimes q^{-h_0/2}) = (q^{h_0/2} \otimes e_0)\sigma \tag{6.4}$$

is given by taking the $x \to \infty$ limit of $\check{R}_{\rm ut}(x)$

$$\sigma = \check{R}_{\rm ut}(\infty) = \varrho_+(\infty)P[+] + \varrho_N(\infty)N + \varrho_-(\infty)P[-]. \tag{6.5}$$

On the other hand, the braid generator can also be obtained by taking the $x \to \infty$ limit of $\check{R}(x)$ in the twisted case:

$$\sigma = \check{R}(\infty) = P[+] - \frac{1-q}{f(q)} \frac{2(q+q^{-1})}{\sqrt{3-2q+3q^2}} N - qP[-]. \tag{6.6}$$

Comparing the above two σs, one gets

$$\varrho_+(\infty) = 1 \qquad \varrho_-(\infty) = -q \qquad \varrho_N(\infty) = -\frac{1-q}{f(q)} \frac{2(q+q^{-1})}{\sqrt{3-2q+3q^2}}. \tag{6.7}$$

Multiply the second equation in (6.3) by $P[+]$ from the left and the resulting equation by $P[-]$ from the right, one obtains

$$\begin{aligned}&(\varrho_+(x)P[+] + \varrho_N(x)N)(xe_0 \otimes q^{-h_0/2} + q^{h_0/2} \otimes e_0)P[-]\\ &\qquad = \varrho_-(x)P[+](e_0 \otimes q^{-h_0/2} + xq^{h_0/2} \otimes e_0)P[-].\end{aligned} \tag{6.8}$$

This equation is simplified upon using the relations

$$\begin{aligned}&P[+](q^{h_0/2} \otimes e_0)P[-] = \left(\frac{\varrho_+(\infty)}{\varrho_-(\infty)}P[+] + \frac{\varrho_N(\infty)}{\varrho_-(\infty)}\right)(e_0 \otimes q^{-h_0/2})P[-]\\ &N(q^{h_0/2} \otimes e_0)P[-] = \frac{\varrho_+(\infty)}{\varrho_-(\infty)}N(e_0 \otimes q^{-h_0/2})P[-]\end{aligned} \tag{6.9}$$

which are derived from (6.4) by multiplying $P[+]$ and N from the left, respectively, and $P[-]$ from the right. The simplified expressions read

$$\begin{aligned}&\left\{\left(1 + x\frac{\varrho_+(\infty)}{\varrho_-(\infty)}\right)\varrho_-(x) - \left(x + \frac{\varrho_+(\infty)}{\varrho_-(\infty)}\right)\varrho_+(x)\right\} P[+](e_0 \otimes q^{-h_0/2})P[-] = 0\\ &\left\{\frac{\varrho_N(\infty)}{\varrho_-(\infty)}(x\varrho_-(x) - \varrho_+(x)) - \varrho_N(x)\left(x + \frac{\varrho_+(\infty)}{\varrho_-(\infty)}\right)\right\} N(e_0 \otimes q^{-h_0/2})P[-] = 0.\end{aligned} \tag{6.10}$$

One can easily show that

$$P[+](e_0 \otimes q^{-h_0/2})P[-] \neq 0 \qquad N(e_0 \otimes q^{-h_0/2})P[-] \neq 0. \tag{6.11}$$

It follows that

$$\varrho_-(x) = \frac{x\varrho_-(\infty) + \varrho_+(\infty)}{\varrho_-(\infty) + x\varrho_+(\infty)}\varrho_+(x) = \frac{1-xq}{x-q}\rho_+(x)$$

$$\varrho_N(x) = \frac{\varrho_N(\infty)}{x\varrho_-(\infty) + \varrho_+(\infty)}(x\varrho_-(x) - \varrho_+(x))$$

$$= \frac{1-q}{f(q)} \frac{2(q^2+1)}{\sqrt{3-2q+3q^2}} \frac{(x-1)(x+1)}{(1-xq)(x-q)}\rho_+(x) \tag{6.12}$$

where (6.7) has been used. Thus

$$\check{R}_{\mathrm{ut}}(x) = P[+] + \frac{1-q}{f(q)} \frac{2(q^2+1)}{\sqrt{3-2q+3q^2}} \frac{(x-1)(x+1)}{(1-xq)(x-q)} N + \frac{1-xq}{x-q} P[-]. \tag{6.13}$$

The R-matrix (6.13) also leads to an integrable model of strongly correlated electrons, which, up to a similarity transformation, is the $\alpha = -\frac{1}{2}$ limit of the model proposed in the second paper of [5].

7. Concluding remarks

We have described the twisted quantum affine superalgebra $U_q[sl(2|2)^{(2)}]$ and obtained the R-matrix $\check{R}(x)$, corresponding to the four-dimensional irrep, which is invariant under $U_q[osp(2|2)]$ where $osp(2|2)$ is the fixed-point subsuperalgebra under the automorphism on $sl(2|2)$. This leads to a new four-state model of strongly correlated electrons for which the local Hamiltonian was determined explicitly. It has $U_q[osp(2|2)]$ invariance and the model is exactly solvable in one dimension via the QISM. It is interesting that in the classical ($q \to 1$) limit, the R-matrix admits $sl(2|2)$ invariance and the corresponding exactly solvable model reduces to that Essler *et al* [4].

It was moreover shown that the underlying four-dimensional irrep also gives rise to another $U_q[osp(2|2)]$ invariant R-matrix associated with the untwisted quantum affine superalgebra $U_q[osp(2|2)^{(1)}]$. This R-matrix was determined explicitly and also determines a four-state model of strongly correlated electrons, exactly solvable in one dimension. This latter model in fact arises as the $\alpha = -\frac{1}{2}$ limit of the model proposed in [5].

The R-matrices determined in this paper exhibit the novel feature of having a $U_q[osp(2|2)]$ invariant nilpotent component. They give rise to a local Hamiltonian for a quantum spin chain which is not Hermitian, but nevertheless admits real eigenvalues (for parameters in the appropriate range). This arises due to the fact that in the reduction of the tensor product of the four-dimensional irrep with itself into $U_q[osp(2|2)]$ modules, an indecomposable occurs. New techniques are thus required for the solution of the corresponding Jimbo equations, as we have shown in the paper. Our approach yields a new extension of the twisted tensor product graph method introduced in [9].

The R-matrices, and corresponding exactly solvable models, investigated above are in fact the simplest in an infinite hierarchy arising from the twisted quantum affine superalgebra $U_q[sl(m|n = 2k)^{(2)}]$. Such R-matrices all admit $U_q[osp(m|n)]$ invariance and give rise to new supersymmetric lattice models, exactly solvable in one dimension. Their study is thus of great interest, and it is expected that the novel features observed in the case studies of this paper, will also occur in general. It is hoped that the techiniques we have introduced will provide a basis for the explicit determination of these more general R-matrices and their corresponding lattice models.

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